

Efficient Estimation of the Functional Failure of Natural Circulation System of XAPR based on Data Mining Technology

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ABSTRACT

The Monte Carlo Simulation based method requires considerable computational efforts for the estimation of functional reliability analysis. Efficient sampling techniques can be adopted for performing robust estimations with limited number of samples and associated with computational time. In order to solve the problem of multi-dimensional uncertainties and small functional failure probability in passive system reliability analysis, an innovative optimized line sampling was presented. In the presented method, genetic algorithm was employed to solve the nonlinear constrained minimization problem for identifying the optimal important direction of line sampling, and then the failure probability can be evaluated by line sampling with high efficiency. Taking the reliability estimation on the capacity of natural circulation cooling in reactor core of Xi ' an Pulsed Reactor for example, the uncertainties related to the model and the input parameters were considered in this paper. Natural circulation functional failure probability was calculated under middle loss of coolant accident (MLOCA). The numerical results show that the presented method has the high computing efficiency and excellent computing accuracy compared with traditional probability analysis methods. In addition, this method demonstrates the efficiency and feasibility for functional reliability analysis of implicit nonlinear limit state function with high dimensional variables and small failure probability.

Keywords: probabilistic safety assessment; functional failure; Efficient Estimation; natural circulation; Optimized Line Sampling Method; Xi'an Pulsed Reactor

1. INTRODUCTION

As a pool-type research reactor, Xi 'an Pulse Reactor (XAPR) has the non-dynamic safety after shutdown. In the event of reactor accident, the core residual heat is inactivated by the natural circulation cooling of the reactor pool water[1]. However, the B-type passive system [2] is easy to be affected by uncertainty in order to alleviate serious accidents, which leads to the failure of the system[3]. Therefore, it is necessary to study the functional reliability of XAPR core's natural cycle cooling capacity and develop a method to evaluate the reliability to provide important information for XAPR Probabilistic Safety Assessment (PSA).

In recent years, Michel Marques [4] and Jafari [5] have put forward functional reliability evaluation methods Reliability Methods for Passive Safety (RMPS) and Reliability Evaluation of Passive Safety (REPAS) respectively, which are used in two-phase flow natural circulation system. In China, Xie Guofeng [6] and Tong Jiejuan [7] are used to calculate the failure probability of the residual heat removal system of high temperature gas cooled reactor by the Response Surface method (RS) and Monte Carlo Simulation (MCS) method respectively. Xia Shaoxiong [8] and Pan Xiaolei [9] respectively use Neural Network Method (NNM) and RMPS to study the reliability of non-dynamic

system in China lead-based research. The results show that MCS, reduced variance MCS and RS method can be used in functional reliability evaluation. However, the above methods need to do a lot of sampling for the small function failure probability, and the calculation efficiency is not high[10].

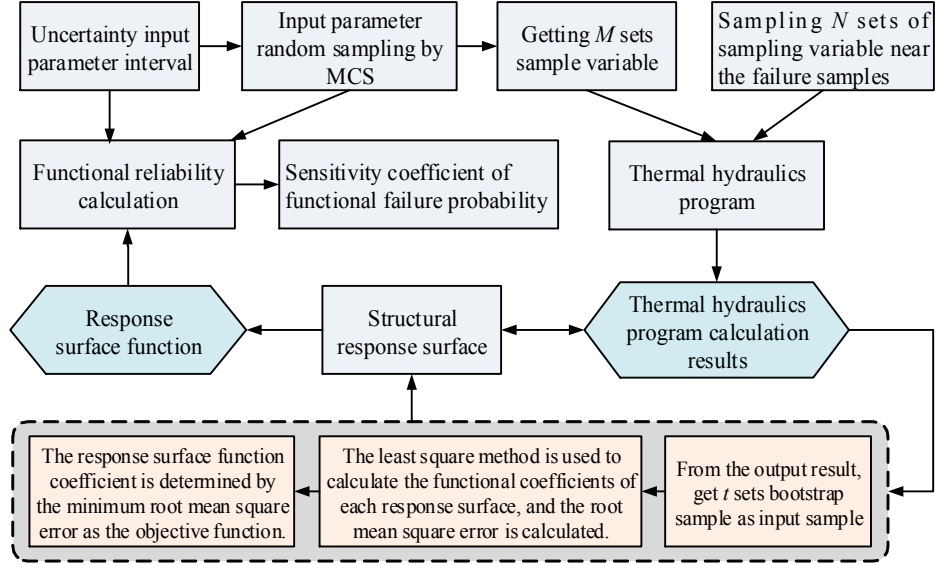


Fig.1 Flowchart of improved response surface method

In this paper, an improved simulation approach called as Line Sampling (LS), which can efficiently evaluate reliability in such a high dimensional problems of structural reliability analysis, is adopted[11]. The basic idea is to employ lines, instead of random points, to probe the failure region of multidimensional setting. An important direction α is optimally determined to point towards the failure region F and a number of conditional, one-dimensional problems are solved along such direction, in place of the high-dimensional problem [12]. However, two main issues are still under research: first, long computer times are necessary for each run of thermal hydraulic code; second, the important direction is not east to be searched as the limit sate function is implicit, which requires additional runs resulting in computational time increasing. This work is an attempt to resolve the two main issues to improve computation efficiency for the functional reliability analysis of passive systems by first: a fast-running, surrogate model called improved response surface (IRS) in replace of the long-running thermal hydraulic code; second, an optimal important direction based on Markov Chain procedure for determining the important direction in the failure region. The advantage of this method has been demonstrated and applied in a functional reliability analysis of a passive residual heat removal system.

A sensitivity analysis, which concerns with ranking of the individual uncertainty parameters according to their relative contribution on the functional failure probability, has been carried out. In this case, an alternative approach is applied to identify and rank influential individual uncertainty parameters based on the sensitivity of the cumulative distribution function (CDF) of the functional failure probability. The approach doesn't assume a linear or other explicit functional relationship between the response and the input parameters, and provide more information than the traditional regression-based methods. The sensitivity coefficient is expressed as an expectation of the partial derivative of the failure probability with respect to the distribution parameter.

2. FUNCTIONAL RELIABILITY EVALUATION MATHEMATICAL MODE

Different methods can be used to quantify the passive system functional failure probability once a best estimate thermal hydraulic code and a model of the system are given. Let $\mathbf{X}=(x_1, x_2, \dots, x_n)$ be the vector of the parameters or variables, $y(\mathbf{X})$ be the identified safety variable and A_Y be the permissible value based on failure criterion. Thus, introducing the Performance Function (PF) as $g(\mathbf{X})=A-y(\mathbf{X})$, failure occurs if $g(\mathbf{X})<0$. In this representation, the probability P_f of system functional failure can be evaluated by the following integral:

$$P_f = \int \cdots \int_{g(\mathbf{X})<0} f_{\mathbf{X}}(\mathbf{X}) dx_1 \cdots dx_n \quad (1)$$

where $f_{\mathbf{X}}(\mathbf{X})$ is the joint probability density function of random variable and the integration is carried out over the failure region where $g(\mathbf{X})<0$. Since, $g(\mathbf{X})$ with a thermal hydraulic code is implicit and computationally expensive, approximation to the $g(\mathbf{X})$ is used to bring down the computational cost.

For the convenience of calculation, the formula (1) can be rewritten in the following form:

$$P_f = \int \cdots \int_{\Omega} I[g(\mathbf{X})] f_{\mathbf{X}}(\mathbf{X}) dx_1 \cdots dx_n \quad (2)$$

where $I[g(\mathbf{X})]$ is the indicator function. If $g(\mathbf{X})<0$, $I[g(\mathbf{X})]=1$. if $g(\mathbf{X})\geq 0$, $I[g(\mathbf{X})]=0$. Ω is the whole integral region.

In practice, the multidimensional integral (2) can't be easily evaluated for the passive natural circulation system. Since, $g(\mathbf{X})$ is a complex thermo-hydraulics program, the functional function is a simulation program, which needs to be solved by numerical method. Because of the small probability of functional failure, it is necessary to make multiple sampling by MCS calculation, and each sampling should run the thermal-hydraulic program, the calculation efficiency is low. Some efficient simulation techniques have been used for this purpose. In this paper, in order to improve computational efficiency two methods are proposed: firstly, a response surface replacement regression model is established to speed up the computation speed; secondly, the efficient MCS method named as an optimized line sampling (OLS) method is used to obtain more robust estimation results with less operation times.

3. PASSIVE SYSTEMS FUNCTIONAL FAILURE RELIABILITY ANALYSIS METHODOLOGY

3.1 Improved Response Surface (IRS) method

The natural cycle process of XAPR core is simulated by thermal hydraulics program. Its functional function is implicit, and the failure response surface can be constructed to approximate the implicit functional function [6]. If it is solved by the RS method directly, it is necessary to run the thermal hydraulic program $k(2n + 1)$ times[9]. In order to reduce the number of calculations, a multi-dimensional fitting method is proposed to construct the functional function of the failure response surface, which is used instead of the thermal hydraulic program. The IRS process is shown in Fig. 1. The essential steps are as follows[13]:

(1) Getting the sample sets: The sample sets in this paper are divided into two parts. Firstly, M sets input vectors are randomly simulated by direct MCS in the uncertainly input parameter interval, and the corresponding output is calculated by the thermal hydraulic program. Secondly, the important density function is used to sample N sets of

conditional sample vectors near the failure region, and the corresponding output is calculated by the thermal hydraulic program.

(2) Estimating the coefficients of the sets of response surface function: Using bootstrap method to generate t sets bootstrap self-lifting samples from the samples obtained from step 1) as input samples, and the corresponding output is calculated by the thermal hydraulic program. The least square method is used to calculate the function coefficient of each response surface, and the root mean square error is calculated. The response surface function coefficient is determined by taking the minimum root mean square error as the objective function.

(3) Analyzing the functional failure probability: Sampling q sets of input vectors from the distributions of the uncertainty input variables by IRS method. And the cumulative distribution function and confidence interval of the system output response are obtained from the response surface function of step 2. Then, the functional failure probability of the passive system is calculated statistically.

3.2 Methodology for the optimized Line Sampling

LS is a stochastic method for efficiently estimating multi-dimensional and low failure probability, originally developed for the reliability analysis of structural systems[11][12]. The efficiency depends on the determination of the important direction. Unfortunately, the important direction is not easy to be searched as the limit state function is implicit. The key steps of the method concern with the searching of the optimal important direction and the drawing of the samples for failure probability estimation. In this work, these two key steps are carried out with the aid of the samples distributed in the failure region by Markov Chain simulation, which are used both to search for the optimal important direction and to evaluate the failure probability.

The optimal important vector α^{opt} is computed as the normalized “center of mass” of failure region. A point X_0 is taken in the failure region: Subsequently, X_0 is used as the initial point of a Markov Chain which lies entirely in the failure region. A Metropolis-Hastings algorithm is employed to generate a sequence of N_s points lying in the failure region. Markov Chain simulation can accelerate the efficiency of exploring the failure region. The vectors $X_j / \|X_j\|$ are then averaged to obtain the important direction α . The direction provides a good “map” approximating of the optimal important direction α^{opt} of failure region. Thus, it provides in principle the most realistic and reliable estimate for the important direction. The process is given by steps as following (Fig. 2):

(1) Select X_0 as the initial value of Markov Chain: X_0 should be in the failure region, which is determined by traditional MC sampling or engineering judgment.

(2) Generate $(k+1)$ th $\tilde{X}_{k+1} = (x_1^{k+1}, x_2^{k+1}, \dots, x_j^{k+1}, \dots, x_n^{k+1})$: Let $X_k = (x_1^k, x_2^k, \dots, x_j^k, \dots, x_n^k)$ be the k th Markov Chain sample and let $p_j^*(\xi_j | x_j^k)$ $j=1,2,\dots,n$ be a one-dimensional \mathcal{E}_j^{k+1} from $p_j^*(\cdot | x_j^k)$ based on the current sample value x_j^k . Let $q(X) = \prod_{j=1}^n q_j(x_j)$ be the PDF of the variable X , where $q_j(x_j)$ denotes the one-dimensional PDF of x_j . Then, the ration $r_j^{k+1} = q_j(\xi_j^{k+1}) / q_j(x_j^k)$ is computed. Finally, the j th component of X_{k+1} is accepted as following:

$$\tilde{x}_j^{k+1} = \begin{cases} \xi_j^{k+1} & \text{with probability } \min(1, r_j^{k+1}) \\ x_j^k & \text{with probability } 1 - \min(1, r_j^{k+1}) \end{cases} \quad (3)$$

(3) Accept/reject of the candidate sample \tilde{X}_{k+1} : if $\tilde{X}_{k+1} = X_k$, set $X_{k+1} = X_k$. Otherwise, judge whether \tilde{X}_{k+1} belongs to the failure region, if $\tilde{X}_{k+1} \in F$, accept it as the next state, i.e., set $X_{k+1} = \tilde{X}_{k+1}$; otherwise, reject it and take the current state as the next one, i.e., set $X_{k+1} = X_k$.

(4) Repeat steps 2 and 3 until N_s Markov Chain samples $X_j (j=1,2,3,\dots,N_s)$ are generated.

(5) Calculate the average of the unit vectors $X_j/\|X_j\|$ to obtain the optimal important direction $\alpha^{\text{opt}} = \frac{1}{N_s} \sum_{j=1}^{N_s} X_j / \|X_j\|$ and the unit optimal important direction $e_a^{\text{opt}} = \alpha^{\text{opt}} / \|\alpha^{\text{opt}}\|$ of the OLS method, which is shown in Fig. 3 (A).

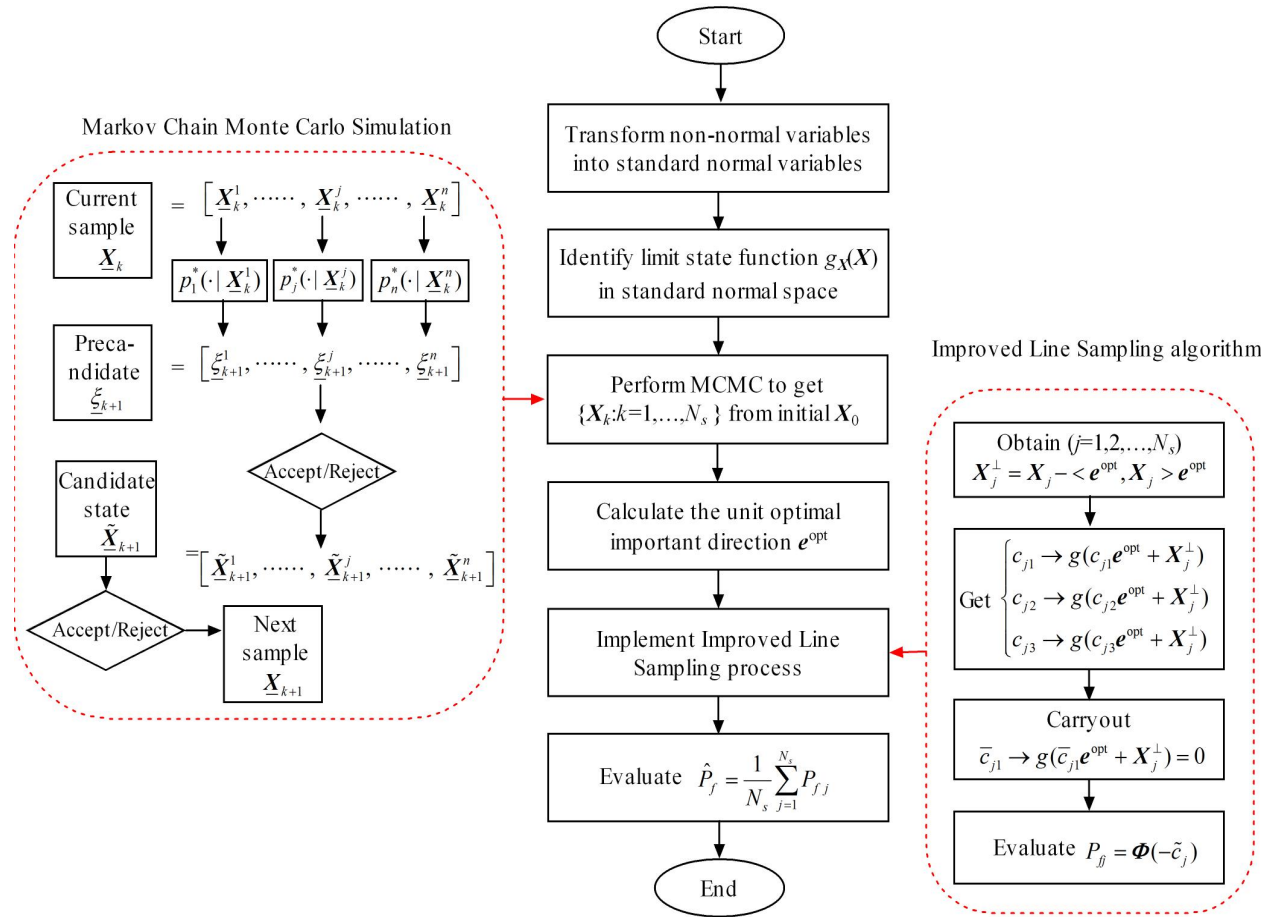


Fig.2 Flow diagram of the improved Line Sampling method

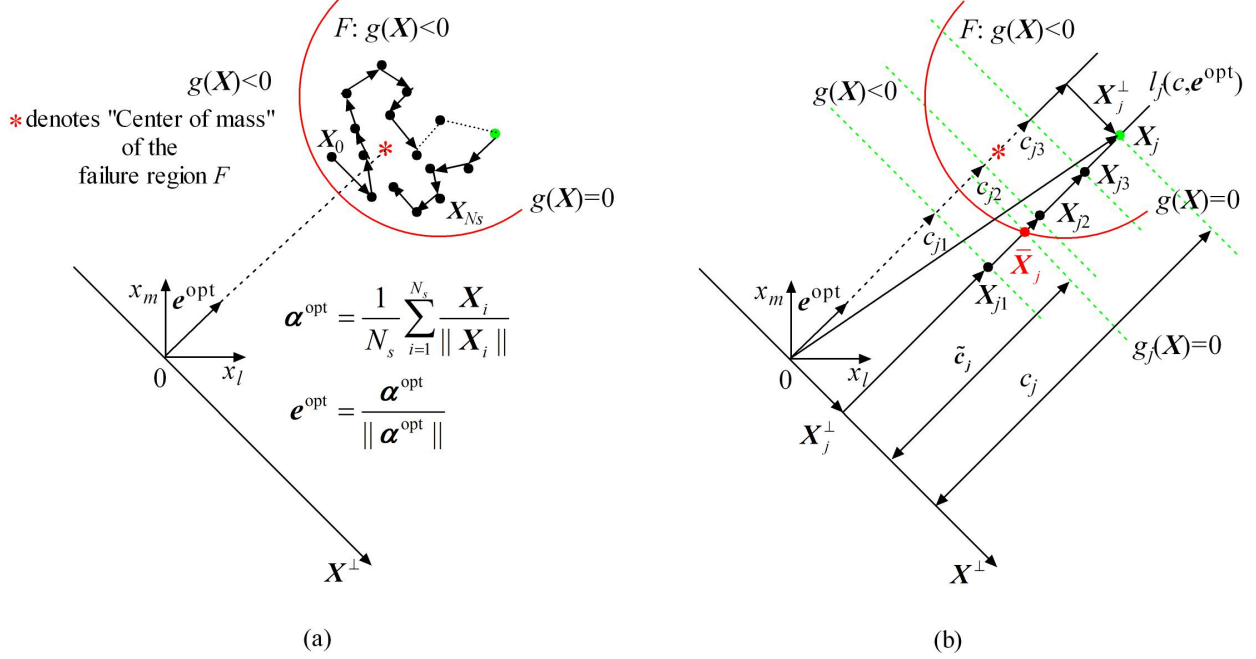


Fig.3 (a) Important direction determined by Markov Chain. (b) Line Sampling randomly selected parallel lines

Once the optimal important direction α^{opt} has been determined, the unit vector e_a^{opt} can be calculated by $e_a^{\text{opt}} = \alpha^{\text{opt}} / \|\alpha^{\text{opt}}\|$. Each Markov Chain sample X_j in the failure region is decomposed into the one-dimensional space $c e_a^{\text{opt}}$ and $(n-1)$ dimensional vector X_j^\perp perpendicular to the unit important direction e_a^{opt} in the standard normal space. The sample vector can be expressed as $X_j = c e_a^{\text{opt}} + X_j^\perp$, where $X_j^\perp = X_j - \langle e^{\text{opt}}, X_j \rangle e^{\text{opt}}$, $\langle e^{\text{opt}}, X_j \rangle$ is the scalar product between e_a^{opt} and X_j and c is the real number in $[-\infty, +\infty]$, shown in Fig. 3 (B).

After the decomposition for each sample in the failure region is completed, three different discrete points $c_i e_a^{\text{opt}} + X_j^\perp$ by giving three constant c_i ($i=1,2,3$), i.e. $(X_{j1} = c_{j1} e_a^{\text{opt}} + X_j^\perp, X_{j2} = c_{j2} e_a^{\text{opt}} + X_j^\perp, X_{j3} = c_{j3} e_a^{\text{opt}} + X_j^\perp)$ determine a straight line $l_j(c, e_a^{\text{opt}})$ shown in Fig. 3 (B). By quadratic polynomial interpolation by means of limit state function $g(c e_a^{\text{opt}} + X_j^\perp)$ at three discrete points, the coefficient \tilde{c}_j , which represents the intersection between the limit state function $g(\tilde{c}_j e_a^{\text{opt}} + X_j^\perp) = 0$ and the straight line $l_j(c, e_a^{\text{opt}})$ can be obtained. Then, the conditional one-dimensional failure probability P_{fj} associated to each random sample X_j can be estimated as

$$\hat{P}_{fj} = P[N(0,1) > \tilde{c}_j] = 1 - P[N(0,1) \leq \tilde{c}_j] = 1 - \Phi(\tilde{c}_j) = \Phi(-\tilde{c}_j) \quad (4)$$

Then, the unbiased estimation of failure probability \hat{P}_f can be estimated as following:

$$\hat{P}_f = \frac{1}{N_s} \sum_{j=1}^{N_s} P_{fj} \quad (5)$$

Finally, the variation of the estimator can be written as

$$\sigma^2(\hat{P}_f) = \frac{1}{N_T(N_T - 1)} \sum_{k=1}^{N_T} (\hat{P}_{fj} - \hat{P}_f)^2 \quad (6)$$

Notice that the variance of the estimator of failure probability is considerably reduced by the OLS method.

4. FUNCTIONAL RELIABILITY ANALYSIS OF THE XAPR CORE NATURAL CIRCULATION

4.1 System safety function

XAPR is a pool reactor structure with a natural circulation cooling core. When the reactor is shut down, the residual heat of the core can be cooled by the natural circulation of the pool water, and the heat exchange between the pool water and the environment can be used to remove the core residual heat. When LOCA occurs, the accident process can be divided into three stages: 1) water-cooled natural circulation stage; 2) core semi-exposed stages; 3) air-cooled natural circulation stage. In this way, the long-term natural circulation cooling of the core can be established under LOCA, and the residual heat of the core can be removed to ensure the integrity of the fuel element.

4.2 System modeling and verification

The core natural circulation cooling process under LOCA is simulated with RELAP5. The calculation node of XAPR thermal hydraulic model is shown in Fig. 4. The control nodes 101-107 are denoted the reactor pool. The control nodes 120-124 are denoted the air boundary of the reactor pool concrete wall outside. The control nodes 150-163 are denoted the primary pipe. The control nodes 170-172 are denoted the secondary cooling system. The control node 109 is denoted the air boundary of the reactor pool top. In order to verify the accuracy of the model, the steady state operation condition is used as the boundary condition to calculate the natural circulation flow rate of the core is 12.21 kg/s. The flow rate is basically consistent with the actual natural circulation flow of 12.13 kg/s.

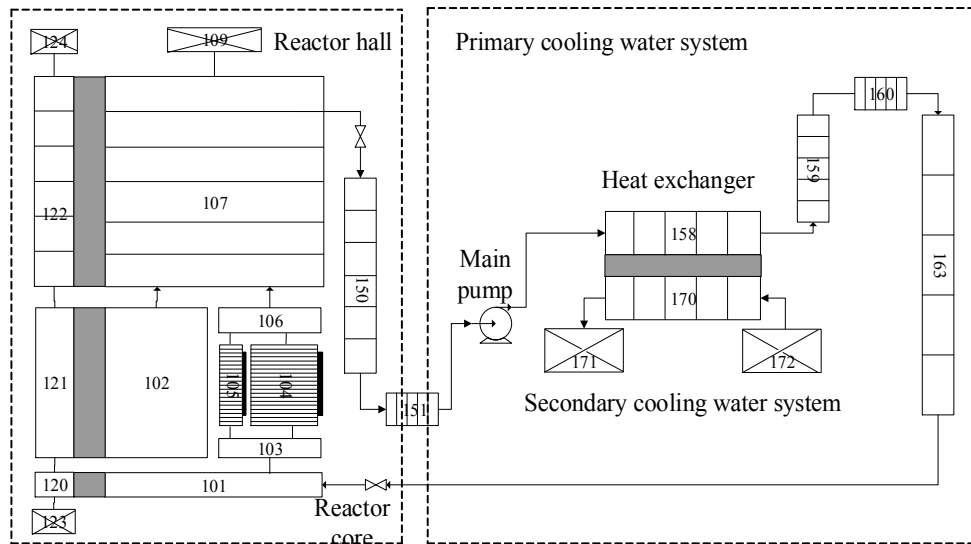


Fig.4 Nodalization of RELAP5 calculation node

4.3 System modeling and verification

Uncertainties in thermal hydraulic conditions can impact the performance of the passive safety systems. To effectively model these uncertainties, it is useful to separate the two kinds of uncertainty, e.g. “aleatory” and “epistemic” [9]. Because of the difference in their fundamental nature, these two types of uncertainties must be considered separately. Aleatory uncertainties concern, for instance, the variability in the actual geometrical properties (due to difference between the as-build system and its design upon which the analysis is based), material properties

(affecting the failure modes) and the stochastic initial/boundary conditions (the actual values taken by design parameters). Epistemic uncertainties arise mainly from the lack of knowledge about the system performance and the conditions in which the phenomena occur, e.g. natural circulation. This uncertainty affects the model representation of the system behavior, in terms of both model uncertainty in the hypotheses and parameter uncertainty in the variations of the operation parameters. In this present analysis for the estimation of the functional failure probability of the passive system, aleatory uncertainties aren't considered.

In this paper, only epistemic like the model and parameter uncertainties are considered and propagated through the thermal hydraulic program code. A tentative list of fourteen uncertain input parameters are identified by Analytic Hierarchy Process (AHP). In the present treatment, the nature of probability distributes is determined from the information on variability from experimental data whenever they are available, otherwise ex-pert/engineering judgment is used.

Table 1 Probability distributions and characteristic parameters of input parameters

Uncertainty	Input parameters	Distribution	Mean	Standard deviation	Interval
Thermal parameter					
	1. The reactor residual heat power deviation Q , %	Normal	1	0.06	0.80-1.20
	2. The inlet temperature of pool water T_w , °C	Normal	35	3.65	29-41
	3. The air inlet temperature T_g , °C	Normal	25	9.12	10-40
	4. The system pressure P , MPa	Normal	0.17	0.02	0.14-0.20
	5. The single-phase heat transfer model factor ξ_1	Normal	1	0.05	0.92-1.08
	6. The two-phase heat transfer model factor ξ_2	Normal	1	0.15	0.75-1.25
Hydraulic parameter					
	7. The single-phase friction model factor ξ_3	Normal	1	0.01	0.98-1.02
	8. The two-phase friction model factor ξ_4	Normal	1	0.10	0.83-1.17
	9. The surface roughness of pipe channel dx	Uniform	2		0-4
	10. The flow inlet resistance coefficient k_{in}	Uniform	50		0-100
	11. The flow outlet resistance coefficient k_{out}	Uniform	50		0-100
	12. The makeup volume V , %	Normal	1	0.06	0.9-1.1
Process parameter					
	13. The shutdown delay time t_d , s	Lognormal	0.6	0.24	0.6-1.0
	14. The fill water tank open delay time t_m , s	Lognormal	40	6.06	30-50
	15. The pump open delay time t_p , s	Lognormal	45	9.09	30-60

In this study, the epistemic uncertainties probability distributions are assumed normal or log-normal with mean values equal to the parameter nominal values. Finally, the fourteen uncertain input parameters considered in the analysis are summarized in Table 1. Then, the thermal hydraulic program code gives the response variable of the fuel core maximum temperature as output.

4.4 Failure criteria

The definition of the functional criterion is: when the cladding temperature is below or equal to 500°C, the fuel core maximum temperature should be lower than 1150°C; Above 500°C, it should be below 970°C. Considering the probability safety view, if the fuel core maximum temperature goes beyond the safety limit of 970°C, the functional failure is considered. Thus, let \mathbf{X} denote the vector of fourteen uncertain parameters of Table 1, $T_{o,max}(\mathbf{X})$ denote the fuel core peak temperature. Then, the functional function can be represented as $g(\mathbf{X})=970-T_{o,max}(\mathbf{X})$, if $g(\mathbf{X})<0$, it is considered to be fail.

4.5 propagation of uncertain

Simulation samples of the uncertainties were drawn on basis of their individual probabilistic distribution parameters. And then, the samples were regarded as input parameters to the thermal hydraulic code RELAP5. The code calculated out the coolant outlet temperature taken as output response. In order to avoid the problem of long simulation times of each code run, approximations to the $g(\mathbf{X})$ were used to bring down the computational effort. Response surface model of the quadratic polynomial type replaced the long-running, original thermal hydraulic code RELAP5 to approximate the output from the code. Finally, the functional failure probability can be assessed by performing the improved LS simulation with the response surface.

5. FUNCTIONAL RELIABILITY ESTIMATION AND COMPARISON OF RESULTS

5.1 Construction and verification of IRS

To account for potential non-linear effects, a quadratic polynomial model was selected to construct a response surface. With twenty-nine predictor variables generated with Bucher design method [14] from Table 1, each set of predictor variables value is entered into the T-H program of the natural circulation system model shown in Fig. 4. And a set of response simulation values can be worked out. The samples used to fit the response surface function include: firstly, sampling $M=120$ sets of the random samples by MCS; secondly, sampling $N=24$ sets of conditional samples by the important density function; thirdly, sampling 12 sets of bootstrap samples near the failure domain by bootstrap method. The total calculation time of RELAP5 was 159.8 h, and $t_M = 120.1$ h, $t_N = 27.3$ h and $t_t = 12.4$ h. The coefficients of response surface function are solved by 156 sets of samples. The quadratic polynomial response surface function is then constructed by as follows:

$$\begin{aligned}\tilde{T}_{o,max}(\mathbf{X}) = & 753.87 + 0.53Q + 0.66T_w + 1.72T_g + 0.18P \\ & - 31.19\xi_1 + 1.34e+2\xi_2 - 6.3\xi_3 + 6.21\xi_4 + 0.09dx \\ & + 13.48k_{in} + 12.98k_{out} - 4.79t_d + 8.54t_m + 12.89t_p \\ & - 2.34Q^2 - 2.71e-4T_w^2 - 12.54T_g^2 - 7.12e-4P^2 \\ & + 10.39\xi_1^2 + 7.04\xi_2^2 - 0.47\xi_3^2 - 0.47\xi_4^2 + 5.43e-4dx^2 \\ & - 3.08k_{in}^2 - 3.00k_{out}^2 + 1.22t_d^2 + 5.74t_m^2 - 9.53t_p^2\end{aligned}\quad (7)$$

Fig. 5a compares the values between RELAP5 output and response surface function, and shows that the fitting values of response surface function are in good agreement with the output values of REALP5. Fig. 5b shows that the fitting errors are all at $\pm 6^\circ\text{C}$, and the response surface function can be used as an alternative model of RELAP5 to calculate the failure probability.

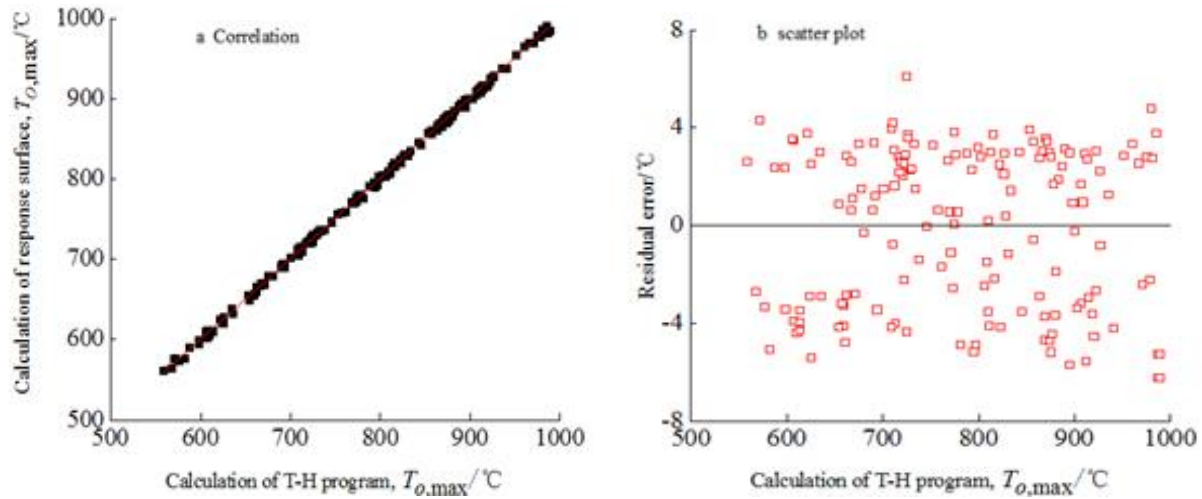


Fig.5 (a) Comparison between code results and reponse surface predictions for the core peak temperature (b) Correlation and residual scatter plot for the fuel core peak temperature

In order to study the sampling efficiency of IRS method, the proportion of 156 sets of samples in RS and IRS was calculated. In 156 sets of samples(Fig.6), about 2 sets of samples outputs $T_{o,max}$ goes beyond 970°C with RS, which accounts for about 1.28%. IRS get 20 sets of samples, the proportion of which is 9.61%, and the probability of failure events is nearly 8 times, and the sampling efficiency was obviously increased. Fig. 7 shows the results of the RS and IRS predictions of $T_{o,max}$ with 20 sets of samples.

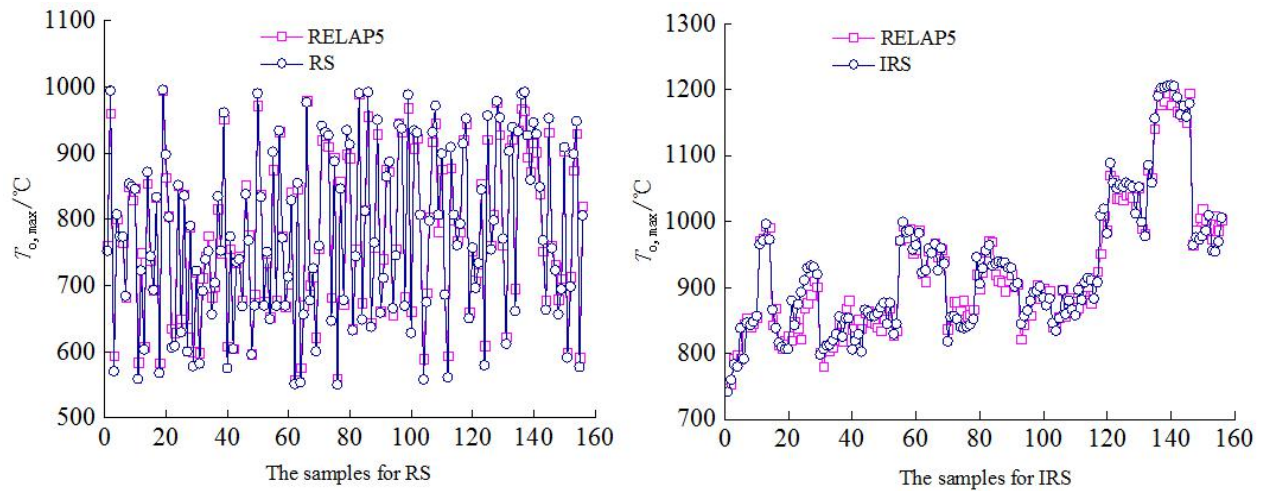


Fig.6 Correlation between RS and IRS Prediction for Fuel Peak Temperature

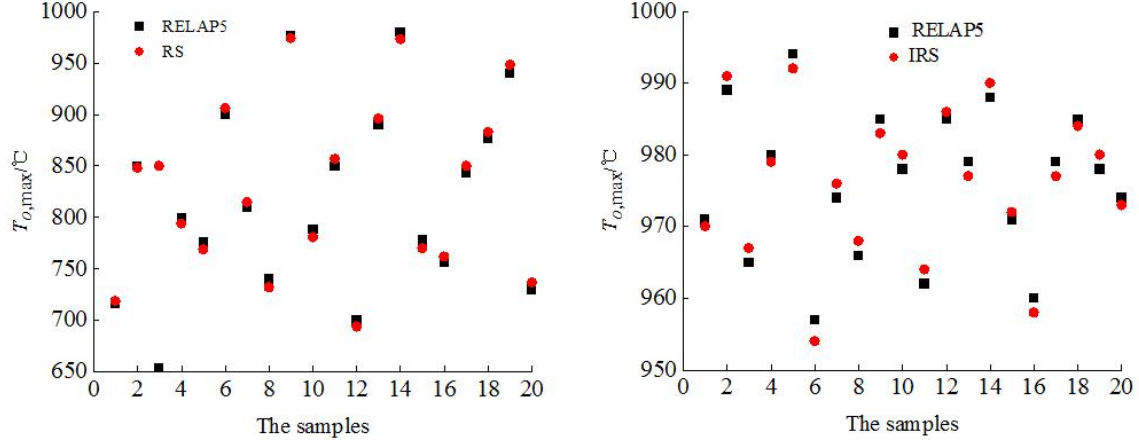


Fig.7 Correlation between RS and IRS prediction for the fuel core peak temperature

5.2 Functional failure probability calculation and comparison

In this study, OLS is compared to direct MCS, Advanced Monte Carlo Simulation (AMCS) and Advanced Importance Sampling (AIS) for the estimation of the functional failure probability for the passive residual heat removal system, respectively. Fig. 8 depicts the functional failure probability convergence with the sampling simulation numbers for four different runs of direct MCS, AMCS, AIS and OLS, respectively. And the abscissa axis is marked by the logarithmic coordinate. It can be seen that when the sampling simulation number is more than 5,000 the functional failure probability values are less sensitive to the sampling simulation number for OLS (red symbol line). However, the outcomes still have the fluctuation even the sampling simulations number arriving at 20,000 for AIS (cyan symbol line). While the sampling simulation number must reach 50,000 (blue symbol line) for AMCS and 100,000 (black symbol line) for direct MCS, respectively, the convergence of failure probability can be accordant. It can be found that IS-SS can reduce the computational simulations and improve the efficiency compared with three other methods. Thus, according to the convergence of four different methods, for our case, OLS has been drawn with a total of $N_T=5,000$ simulations for the estimation of the functional failure probability. Meanwhile, direct MCS with a total of $N_T=100,000$ simulations, AIS with a total of $N_T=20,000$ simulations and AMCS with a total of $N_T=50,000$ simulations have been drawn to estimate the functional failure probabilities of the passive system, respectively.

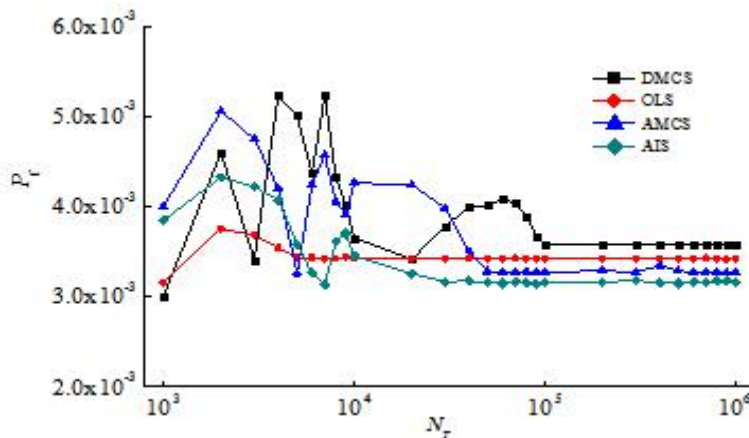


Fig.8 Failure probability convergence with N_T

In this section, the performance of the improved LS (OLS) method in the task of estimating the functional failure probability of the XAPR is compared to that of DMCS method; a comparison is also carried out with respected to the original AIS method. Fig. 9(a) and (b) show and compare the empirical probability density functions (PDFs) and cumulative probability functions (CDFs), respectively, of the coolant outlet temperature $T_{out,core}$ obtained with $N_T=1000,000$ simulations by DMCS through the code RELAP5 (black solid lines). The same figures also show the PDFs and CDFs constructed with $N_T=5,000$ simulations by OLS through RS model (red dashed lines) and $N_T=5,000$ simulations by OLS through RS model (blue dotted lines), respectively. Notice that the results have been obtained with a very large number $N_T=100,000$ simulations of the original code, to provide a robust reference for the comparison. It can be seen that the results obtained from the RS model by OLS are in good agreement with those obtained from the original code by DMCS. Also, it can be seen that the OLS estimators are much closer to the reference values than the AIS estimators.

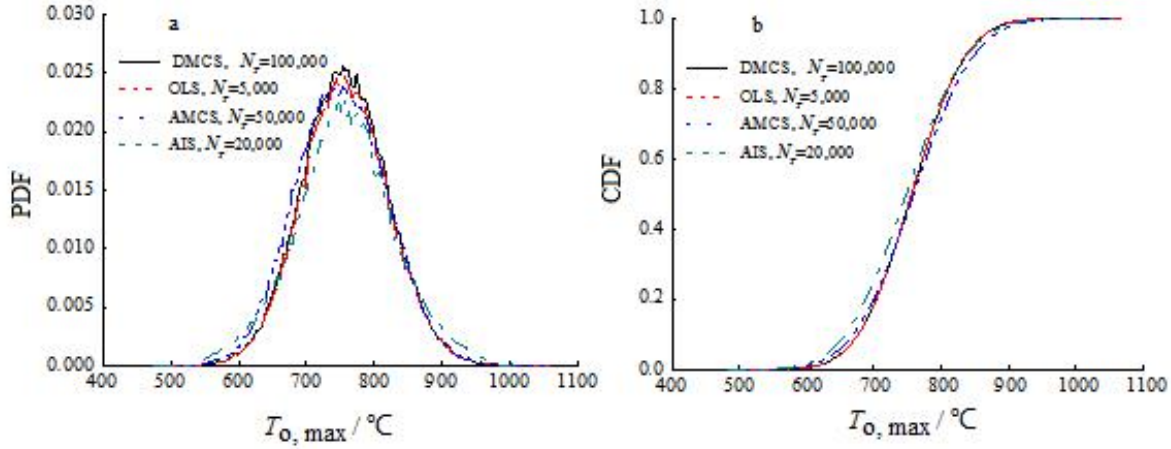


Fig.9 (a) The Fuel core peak temperature empirical PDFs. (b) the fuel core peak temperature empirical CDFs

To compare the computational efficiency for functional failure probability estimations, a numerical index called as the unitary coefficient of variation Δ , which is independent of the total number N_T is introduced. The unitary c.o.v. Δ is defined as $\Delta = \delta[\hat{P}_f] / \sqrt{N_T}$. It is considered that the lower the value of Δ is and the higher the computational efficiency is. Then, in order to compare the computational accuracy, another numerical index known as the relative error ξ of the functional failure probability estimator is introduced as the ratio of the deviation of failure probability to the real failure probability. Notice that the smaller the value of the relative error is, the higher the computational accuracy is.

According to Table 2, it can be seen that the failure probability estimator with a total of 5,000 samples drawn by OLS+IRS can achieve the calculation accuracy with a total of 10^5 samples drawn by DMCS+RELAP5. However, RS is an approximate method of curve fitting. Its calculation precision depends on the linearity of function function, and the function function of XAPR core natural circulation system is nonlinear, so the result is not good. The sampling efficiency by DMCS+RELAP5 is low and the computation time is about 6.25 times as long as OLS+IRS when the

sampling number is 10^4 . In the same sampling number, the computational time by RS is close to OLS+IRS, but its calculation accuracy is poor.

Table 2 Results for different calculation methods

Methods	Number of samples N_T	Failure probability P_f	Δ	Relative error	Computational time/h
DMCS	10^5	3.480×10^{-3}	17.09	NA	1002.8
RS	157	3.197×10^{-3}	3.23	8.13%	159.8
AMCS+RS	50,000	3.125×10^{-3}	10.17	10.2%	687.4
AIS+RS	20,000	3.057×10^{-3}	4.13	12.1%	365.3
OLS+IRS	5,000	3.405×10^{-3}	1.68	2.15%	127.8

5.3 Reliability sensitivity analysis

The sensitivity analysis can reflect the uncertainty of input parameters, which can improve the reliability of the system and reduce the uncertainty effectively. Due to the implicit non-linear relation in the non-dynamic physical process, the traditional correlation coefficient (CCS), the standard regression coefficient (SRCS), the rank correlation coefficient (RCCS) and the standard rank regression coefficient (SRRCS) have some limitations [15].

In this view, the sensitivity of the passive system performance to the individual uncertain input parameters of Table 1 has been studied based on the sensitivity of failure probability with regards to CDF of the response variable. The normalized reliability sensitivity coefficient is expressed as an expectation of the partial derivative of the PDF, evaluated over the failure region; where in the sampling based method can be used to compute the reliability sensitivity. The sensitivity is defined as the partial derivative of the functional failure probability with respect to the distribution parameter of the basic random variable, which can be formulates as:

$$\frac{\partial P_f}{\partial \mu_{x_k}} = \int \dots \int_{\Omega} \frac{\partial f_x(X)}{\partial \mu_{x_k}} dX = \frac{1}{N_T \sqrt{2\pi}} \sum_{i=1}^{N_T} \left[\frac{e_{\alpha_i} \exp(-\tilde{c}_i / 2)}{\mu_{x_k}} \right] \quad (8)$$

where μ_{x_k} denotes the mean value of the uncertainty parameter. The partial derivative $\partial P_f / \partial \mu_{x_k}$ denotes the sensitivity, which is the change in probability due to the change in a distribution parameter. The sensitivity can reflect the significance of the distribution parameter with respect to the failure probability. When $\partial P_f / \partial \mu_{x_k}$ is negative, it implies that the increase of the distribution parameter leads to the decrease of the functional failure probability. Otherwise, the increase of that leads to an increase of the probability.

In order to eliminate dimensional effect, introducing a probabilistic sensitivity coefficient, the mean sensitivity, which is defined as $S_{\mu_{x_i}} = (\partial P_f / \partial \mu_{x_k}) \times (\sigma_{x_k} / P_f)$ [16], where σ_{x_k} denotes the standard deviation.

The normalized sensitivity coefficients (NSC) of the input parameters affecting the behavior of XAPR for the fuel core peak temperature are given in Fig. 10. In can be seen that the decay heat power deviation Q , the air inlet temperature T_g , the reactor pool water inlet temperature T_w , the flow inlet resistance coefficient k_{in} and the pump

open delay time t_p are highly sensitive among others. Among them, Q , T_g and t_p are the most sensitive. Because these input parameters directly affect the heat transfer capacity and the setting time of the natural circulation after the accident, the reliability of the residual heat removal function of the core is greatly affected. Therefore, reducing the uncertainty of the input parameters can effectively reduce the failure probability of the core natural circulation and improve the reliability of the system.

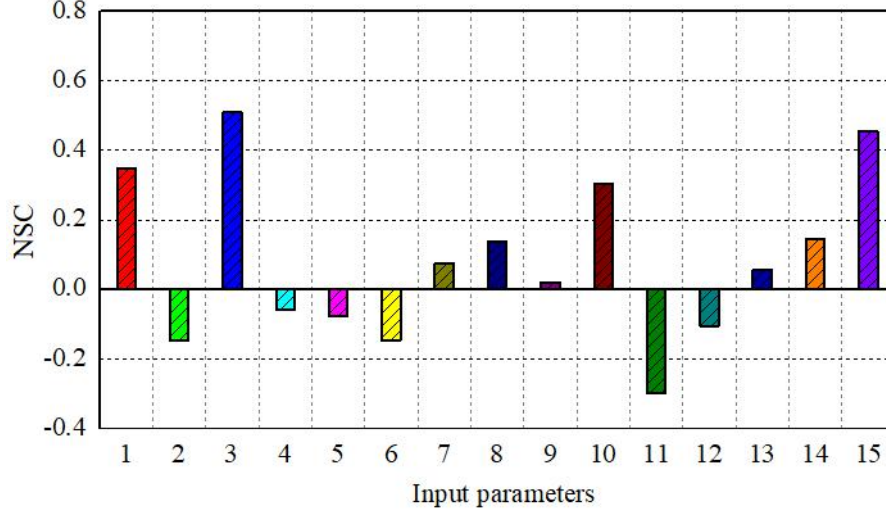


Fig. 10 Normalized sensitivity coefficients of the input parameters

6. CONCLUSIONS

In this paper, the estimation of the functional failure analysis of XAPR reactor core natural circulation cooling system is carried out by sampling the epistemic uncertainties associated with the system model and input parameters, following the presumed conservative initial condition under MLOCA. The Optimized Line Sampling (OLS) method in conjunction with the improved response surface (IRS) method has been utilized to perform an efficient estimation of the functional failure probability. The conclusions are as follows:

(1) The proposed method combines IRS and OLS, and genetic algorithm was employed to solve the nonlinear constrained minimization problem for identifying the optimal important direction of line sampling, and then the failure probability could be evaluated by line sampling with high efficiency. Compared with other methods, this method has higher sampling efficiency and can guarantee better calculation precision, and has strong adaptability to the natural circulation system with higher nonlinear degree.

(2) In the case of MLOCA in XAPR, the reactor core natural circulation cooling failure caused by epistemic limitation uncertainties always has a non-zero probability, and the failure probability is 3.480×10^{-3} . Therefore, the functional failure is important in the reliability evaluation of passive natural circulation, and its failure probability should be evaluated based on thermal hydraulic calculation and reliability method.

(3) It is found that the uncertainty in decay heat power deviation Q , air inlet temperature T_g , reactor pool water inlet temperature T_w , flow inlet resistance coefficient k_{in} and pump open delay time t_p are highly more sensitive

among others. By reducing the uncertainty of the above five key parameters, the failure probability of XAPR reactor core natural circulation can be reduced more effectively and the functional reliability can be improved.

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